Exomple Problems

1) $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $\quad y = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$ $\quad z = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ it is given that x, y, z Lineorly dependent

$A = \begin{bmatrix} a & d & p \\ b & e & q \\ c & f & r \end{bmatrix}$ $\qquad B = \begin{bmatrix} a & b & c \\ d & e & f \\ p & q & r \end{bmatrix}$

Results

rows of A Lineorly dependent

columns " " " "

A is singular

inverse of A does not exist

inverse of B does not exist

B is singulor

$\det A = \det B = 0$

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2) $X = \begin{bmatrix} m \\ n \\ p \end{bmatrix}$ $\quad y = \begin{bmatrix} q \\ r \\ s \end{bmatrix}$ $\quad z = \begin{bmatrix} v \\ w \\ t \end{bmatrix}$ it is given thot x, y, z Lineorly independent

$C = \begin{bmatrix} m & q & v \\ n & r & w \\ p & s & t \end{bmatrix}$ $\qquad D = \begin{bmatrix} m & n & p \\ q & r & s \\ v & w & t \end{bmatrix}$

Results

rows of C are independent

columns of C " "

C^{-1} exists, D^{-1} exists

$C = D^T$

$\det C \neq 0$ $\qquad \det D \neq 0$ $\qquad \boxed{\det C = \det D}$

Example problems

3) Examine the following system of equations

$a \neq 0 \quad b \neq 0$

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 1 & 3 \\ 0 & 8 & 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} a \\ b \\ 0 \end{bmatrix}$$

determine the existence of solution

$$\bar{A} = \begin{bmatrix} 4 & 0 & 0 & 0 & a \\ 0 & 4 & 1 & 3 & b \\ 0 & 8 & 2 & 6 & 0 \end{bmatrix} \xrightarrow{-2R_2 + R_3 \to R_3} \left[\begin{matrix} \underbrace{\begin{matrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{matrix}}_{A} & \begin{matrix} a \\ b \\ -2b \end{matrix} \end{matrix}\right]$$

$\text{rank } A = 2 \qquad \text{rank } \bar{A} = 3$

$\text{rank } A \neq \text{rank } \bar{A}$ No solution

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4) Examine the following equations and determine the existence of solutions

$$\begin{bmatrix} 2 & a & b \\ 0 & 4 & c \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2 & a & b & 4 \\ 0 & 4 & c & 4 \\ 0 & 0 & 7 & 1 \end{bmatrix}$$

$\text{rank } A = \text{rank } \bar{A} = 3 = n$

Unique solution

Dr Ramazan