Analytical Methods in Engineering (EE300) Major1 **Computer ID** Ramazan

(Show your procedures)

1)Examine the following price list

Apple (kg)	Orange (kg)	Pear (kg)	Total Price (SR)
1	0	2	7
0	2	4	16
3	2	10	37

) Write the necessary equations in matrix form.

) Examine the existence of solution. (unique solution, multiple solution, no solution)

$$\begin{bmatrix}1&0&2\\0&2&4\\3&2&10\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\16\\37\end{bmatrix}$$

$-R_1+R_3\to R_3$

$$\left[\begin{array}{ccc|c}1&0&2&7\\0&2&4&16\\0&2&4&16\end{array}\right]\to\left[\begin{array}{ccc|c}1&0&2&7\\0&2&4&16\\0&0&0&0\end{array}\right]$$

rank $A=2$ rank $\bar{A}=2$

multiple solution.

2) It is known that the vectors X,Y are linearly dependent. Calculate p, q.

$$X=\begin{bmatrix}7\\p\\3\end{bmatrix},\quad Y=\begin{bmatrix}0.7\\7\\q\end{bmatrix},$$

$X=10Y$

$p=7\times10=70$

$3=10\times q$

$q=0.3$

$\boxed{p=70 \quad q=0.3}$

3)Fill the blanks

$$x=\begin{bmatrix}2\\2\\0\end{bmatrix},y=\begin{bmatrix}1\\1\\0\end{bmatrix},z=\begin{bmatrix}0\\0\\2\end{bmatrix}$$

x and y are linearly dependent

x and z are linearly Indep

y and z are linearly Indep.

$x=2y \quad \begin{bmatrix}2\\2\\0\end{bmatrix}=2\begin{bmatrix}1\\1\\0\end{bmatrix}$

$z=\alpha x \Rightarrow \begin{bmatrix}0\\0\\2\end{bmatrix}=\alpha\begin{bmatrix}2\\2\\0\end{bmatrix}$ $\quad 2\times\alpha=0$ $\quad \alpha\times0=2$?

4) Examine the following equation systems

$$\begin{bmatrix}3&2&4\\-3&6&1\\3&10&9\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

a)How many nontrivial solutions are there in this equation system

b)Find one nontrivial solution

$$\begin{bmatrix}3&2&4\\0&8&5\\0&8&5\end{bmatrix}\to\begin{bmatrix}3&2&4\\0&8&5\\0&0&0\end{bmatrix}$$

$r=2$

$n=3$

$n-r=3-2=1$ Nont

1 Veriable free.

$\boxed{z=1}$ $\quad 8y+5z=0 \quad y=-\frac{5}{8}$

$3x=-2y-4z=-2\times\left(-\frac{5}{8}\right)-4$

$3x=\frac{5}{4}-4=\frac{-11}{4}$

$x=\frac{11}{12}=0.9$

$\boxed{y=1 \quad z=-\frac{8}{5}}$

$3x=-2(1)-4\times\left(-\frac{8}{5}\right)=$

$x=\frac{22}{15}=$

5) Calculate the inverse of the following matrix by Gaus iteration Method.

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \frac{R_2}{2} \to R_2, \quad R_3 + R_4 \to R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \quad -4R_4 + R_1 \to R_1$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & -4 & -4 \\ 0 & 1 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right]$$

6)Examine the following equation systems
a,b are any **nonzero** number. ($a\neq 0$, $b\neq 0$)

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 1 & 3 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & e \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

d is a nonzero number ($d\neq 0$)

State true or false.

a)if e=0 then this system has multiple solution F

b)if e=0 then this system has no solution T

c)if e≠0 then this system has unique solution T

d)We must know the exact values of a, b,c, in order to say anything F

Explanation required.

7)Examine the following equation systems
a,b,c are any number.

$$\begin{bmatrix} 2 & a & b \\ 0 & 4 & c \\ 0 & 8 & 2c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

State true or false.

a)This system **may** have multiple solution .... F

b)This system has **always** unique solution .... F

c)We cannot say anything unless we know the values of a,b,c .... F

$$\begin{bmatrix} 2 & a & b & 4 \\ 0 & 4 & c & 4 \\ 0 & 8 & 2c & 0 \end{bmatrix} \quad -2R_2 + R_3 \to R_3$$

$$\begin{bmatrix} 2 & a & b & 4 \\ 0 & 4 & c & 4 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

No solution.

Explonotion required.

8)Find the rank of the following matrices

$$A=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad B=\begin{bmatrix} 2 & 2 & 2 & 1 \\ 4 & 4 & 4 & 2 \\ 0.5 & 0.5 & 0.5 & 0.25 \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 11 \\ 0 & 0 & 12 \end{bmatrix}, \quad D=\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

Rank A= 1 Rank B= 1 Rank C= 2 Rank D= 3

$$\begin{bmatrix} 2 & 2 & 2 & 1 \\ 4 & 4 & 4 & 2 \\ 0.5 & 0.5 & 0.5 & 0.25 \end{bmatrix} \quad \begin{matrix} -2R_1 + R_2 \\ \to \\ -0.5R_1 + R_3 \end{matrix} \quad \begin{bmatrix} 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D^T = \begin{bmatrix} 5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 3 \end{bmatrix}$$

D^T is in echolen form

rank D = rank $D^T = 3$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 11 \\ 0 & 0 & 12 \end{bmatrix} \quad -\frac{12}{11}R_2 + R_3 \to R_3 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 11 \\ 0 & 0 & 0 \end{bmatrix}$$

9) Calculate the determinant of the following matrices P,Q,R

$$P=\begin{bmatrix} a & b & 0 \\ 0 & e & 0 \\ 0 & 0 & k \end{bmatrix},\quad Q=\begin{bmatrix} 1 & b & c & d \\ 0 & g & h & k \\ 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 3 \end{bmatrix},\quad R=\begin{bmatrix} a & 0 & 0 \\ x & e & 0 \\ y & 0 & k \end{bmatrix}$$

$a\begin{vmatrix} e & 0 \\ 0 & k \end{vmatrix} = aek$

$\det Q = 0 \quad R_4 = 3R_3$

$\det R = a\begin{vmatrix} e & 0 \\ 0 & k \end{vmatrix} = aek$

10) The linearly dependent vectors X,Y,Z are given as follows

$$X=\begin{bmatrix} a \\ b \\ c \end{bmatrix},\quad Y=\begin{bmatrix} d \\ e \\ f \end{bmatrix},\quad Z=\begin{bmatrix} g \\ h \\ k \end{bmatrix},$$

The matrix Q and vectors M,N,Q are given below

$$P=\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & k \end{bmatrix},\qquad M=[a \;\; d \;\; g],\quad N=[b \;\; e \;\; h],\quad Q=[c \;\; f \;\; k]$$

State True or False

a) det P=0 ...T....

b) P^{-1} exists ....F...

c) The vectors M,N,Q are linearly dependent...T

d) rank P=3 ...F

11) The linearly independent vectors X,Y,Z and matrices P,Q are given as follows

$$X=\begin{bmatrix} a \\ b \\ c \end{bmatrix},\quad Y=\begin{bmatrix} d \\ e \\ f \end{bmatrix},\quad Z=\begin{bmatrix} g \\ h \\ k \end{bmatrix},\quad P=\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & k \end{bmatrix},\quad Q=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$$

State True or False

c) $P^T=Q$ T....... (T: Transpose)

a) det P=0 ...F....

b) det P = det Q ...T....

d) $P=Q^T$ ...T......

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e) $P^{-1} = Q^{-1}$ ....F..

f) $P^{-1} = [Q^{-1}]^T$ T

g) $P^{-1} = [Q^T]^{-1}$ T