Determinants

Rules:

Dr- Ramazan

1) Exchange of two rows multiplies the value of determinant by (-1)

2) Addition of multiple of a row to another row does not affect the value of determinant

3) If you multiply a row by a constant α, the value of determinant will be multiplied by α.

4) if you multiply a matrix by α the value of determinant will be multiplied by α^n. n is the matrix dimension.

5) if rows are linearly dependent the determinant value is zero

6) columns operation have the same effect as row operations.

6.1) Exchange of columns multiplies det. by (-1)

6.2) Addition of multiple of a column to another column does not affect the det.

6.3) multiplying a column by α, multiplies the det by α)

6.4) if columns are linearly dependent det. is zero.

Dr Ramazan

calculate the determinant of the following matrix.

$$A = \begin{bmatrix} 2 & -3 & 4 & 1 \\ 1 & 1 & 3 & 4 \\ 1 & -4 & 1 & -3 \\ 5 & 0 & 7 & 4 \end{bmatrix}$$

$$\det A = 2\begin{vmatrix} 1 & 3 & 4 \\ -4 & 1 & -3 \\ 0 & 7 & 4 \end{vmatrix} - (-3)\begin{vmatrix} 1 & 3 & 4 \\ 1 & 1 & -3 \\ 5 & 7 & 4 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 & 4 \\ 1 & -4 & -3 \\ 5 & 0 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 3 \\ 1 & -4 & 1 \\ 5 & 0 & 7 \end{vmatrix}$$

$$\det\begin{vmatrix} 1 & 3 & 4 \\ -4 & 1 & -3 \\ 0 & 7 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & -3 \\ 7 & 4 \end{vmatrix} - (-4)\begin{vmatrix} 3 & 4 \\ 7 & 4 \end{vmatrix} + 0\begin{vmatrix} 3 & 4 \\ 1 & -3 \end{vmatrix}$$

$$= 1 \times 25 + 4 \times (-16) + 0 = -39$$

$$\det\begin{vmatrix} 1 & 3 & 4 \\ 1 & 1 & -3 \\ 5 & 7 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & -3 \\ 7 & 4 \end{vmatrix} - 3\begin{vmatrix} 1 & -3 \\ 5 & 4 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 \\ 5 & 7 \end{vmatrix}$$

$$= 1 \times 25 - 3 \times 19 + 4 \times 2 = -24$$

$$\det\begin{vmatrix} 1 & 1 & 4 \\ 1 & -4 & -3 \\ 5 & 0 & 4 \end{vmatrix} = -1\begin{vmatrix} 1 & -3 \\ 5 & 4 \end{vmatrix} + (-4)\begin{vmatrix} 1 & 4 \\ 5 & 4 \end{vmatrix} - 0\begin{vmatrix} 1 & 4 \\ 1 & -3 \end{vmatrix}$$

$$= -19 + (-4) \times (-16) - 0 = 45$$

$$\det\begin{vmatrix} 1 & 1 & 3 \\ 1 & -4 & 1 \\ 5 & 0 & 7 \end{vmatrix} = -1\begin{vmatrix} 1 & 1 \\ 5 & 7 \end{vmatrix} + (-4)\begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} + 0\,|\;\;|$$

$$= -1(2) + (-4)(-8) + 0 = 30$$

$$\det A = 2 \times (-39) - (-3)(-24) + 4(45) - 1(30) = 0$$

Dr Ramazan

Calculate the determinant of the following matrix

$$A = \begin{bmatrix} 2 & -3 & 4 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & -2 & 4 & 2 \\ 7 & 6 & 10 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & -2 & 4 & 2 \\ 7 & 6 & 10 & 6 \end{bmatrix} \begin{matrix} -0{,}5R_1 + R_2 \to R_2 \\ -1{,}5R_1 + R_3 \to R_3 \\ -3{,}5R_1 + R_4 \to R_4 \end{matrix} \begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2{,}5 & 1 & 3{,}5 \\ 0 & 2{,}5 & -2 & 0{,}5 \\ 0 & 16{,}5 & -4 & 2{,}5 \end{bmatrix} \begin{matrix} -R_2 + R_3 \to R_3 \\ -\frac{16{,}5}{2{,}5} R_2 + R_4 \to R_4 \end{matrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2{,}5 & 1 & 3{,}5 \\ 0 & 0 & -3 & -3 \\ 0 & 0 & -10{,}6 & -20{,}6 \end{bmatrix} \quad -\frac{10{,}6}{3} R_3 + R_4 \to R_4 \quad \begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2{,}5 & 1 & 3{,}5 \\ 0 & 0 & -3 & -3 \\ 0 & 0 & 0 & -10 \end{bmatrix} = B$$

B is obtained from A by row operations. So $\det(A) = \det(B)$

$$\det(B) = 2 \begin{vmatrix} 2{,}5 & 1 & 3{,}5 \\ 0 & -3 & -3 \\ 0 & 0 & -10 \end{vmatrix} = 2 \times 2{,}5 \times \begin{bmatrix} -3 & -3 \\ 0 & -10 \end{bmatrix} = 2 \times 2{,}5 \times (-3) \times (-10)$$

$$= 150$$

Result $\underline{\det A = \det B = 150}$

Dr Roman

calculate the determinant of the following matrices

$$A=\begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix} \qquad B=\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix} \qquad C=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$D=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 6 & 0 & 0 \end{bmatrix} \qquad E=\begin{bmatrix} 0 & 3 & 0 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad f=\begin{bmatrix} 2 & 3 & 4 \\ 0 & 5 & 0 \\ 6 & 7 & 8 \end{bmatrix}$$

Solution

$$\det A = 2\begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix} = 48$$

$$\det B = 1\begin{vmatrix} 4 & 0 \\ 5 & 6 \end{vmatrix} = 24$$

$$\det C = 1\begin{vmatrix} 0 & 2 \\ 4 & 5 \end{vmatrix} = -8$$

$$\det D = 3\begin{vmatrix} 4 & 5 \\ 6 & 0 \end{vmatrix} = -90$$

$$\det E = -3\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} = -3(-6) = 18$$

$$\det f = 5\begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix} = 5(-8) = -40$$

Dr Romazen

Calculate the determinant of the following matrices

$$A = \begin{bmatrix} 1 & 10 & 3 \\ 2 & 20 & 4 \\ 3 & 30 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 5 & 7 \\ 3 & 9 & 12 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & \cancel{3}4 & 5 \end{bmatrix}$$

$\det A = 0$ Because column 2 $= 10 *$ column 1

$\det B = 0$ Because $R_3 = R_1 + R_2$

$\det C = 0$ Because $R_2 = 2R_1$

Dr Ramazan

① $A=\begin{bmatrix}0&1&2\\0&0&4\\6&7&8\end{bmatrix} \xrightarrow[(-1)]{\text{row exchange}} \begin{bmatrix}6&7&8\\0&0&4\\0&1&2\end{bmatrix} \xrightarrow[(-1)]{\text{row exchange}} \begin{bmatrix}6&7&8\\0&1&2\\0&0&4\end{bmatrix}=C$

$\det C = 6\times 1\times 4 = 24$

$\det A = \det C \times(-1)\times(-1) = \det C = 24$

② $A=\begin{bmatrix}0&1&2\\3&4&5\\6&8&15\end{bmatrix} \xrightarrow[(-1)]{\text{row ex.}} \begin{bmatrix}3&4&5\\0&1&2\\6&8&15\end{bmatrix} \xrightarrow[\text{row operations}]{-2R_1+R_3\to R_3} \begin{bmatrix}3&4&5\\0&1&2\\0&0&5\end{bmatrix}=C$

$\det C = 3\times 1\times 5 = 15$ $\det A = \det C \times(-1) = -15$

③ $A=\begin{bmatrix}2&3&1&3\\4&6&12&9\\2&3&6&4\\6&13&10&17\end{bmatrix}$ $\begin{matrix}-2R_1+R2\to R_2\\-R_1+R_3\to R_3\\-3R_1+R_4\to R_4\end{matrix}$ $\begin{bmatrix}2&3&1&3\\0&0&10&3\\0&0&5&1\\0&4&7&8\end{bmatrix} \xrightarrow[(-1)]{\text{row exchange}} \begin{bmatrix}2&3&1&3\\0&4&7&8\\0&0&5&1\\0&0&10&3\end{bmatrix}$

$-2R_3+R_4\to R_4$ $\begin{bmatrix}2&3&1&3\\0&4&7&8\\0&0&5&1\\0&0&0&1\end{bmatrix}=C$

$\det C = 2\times 4\times 5\times 1 = 40$ $\det A = \det C\times(-1) = -40$

④ $A=\begin{bmatrix}0&3&0\\1&4&0\\6&5&7\end{bmatrix} \xrightarrow[(-1)]{\text{column exchange}} \begin{bmatrix}3&0&0\\4&1&0\\5&6&7\end{bmatrix}=C$

$\det C = 3\times 1\times 7 = 21$

$\det A = \det C\ (-1) = -21$

⑤ $A=\begin{bmatrix}1&2&0\\2&4&6\\3&11&10\end{bmatrix} \xrightarrow{-2C_1+C_2\to C_2} \begin{bmatrix}1&0&0\\2&0&6\\3&5&10\end{bmatrix} \xrightarrow[(-1)]{\text{column exchanse}} \begin{bmatrix}1&0&0\\2&6&0\\3&10&5\end{bmatrix}=C$

$\det C = 30$

$\det A = -30$