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EE300 - N3

Linear dependence, determinant, Matrix inverse

1- Determinant and inverse of a matrix is defined for square matrices

2- Dimension of a square matrix is number of rows or number of columns

$$A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B=\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Dimension of A is 2

Dimension of B is 3

$$C=\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

det(C) is not defined

C^{-1} is not defined

3.) in a square matrix,

if rows are Linearly dependent, then columns are dependent

if columns " " " " rows " "

4) The following sentences are the same

- rows are Linearly dependent
- columns " " "
- The matrix is singular
- The inverse does not exist
- The determinant is zero
- The rank is less than the dimension

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5) The following sentences are the same D2

- rows are Linearly independent
- Columns " " "
- The matrix is nonsingular
- The inverse exists
- The determinant is not zero
- The rank is equal to the dimension.

6)
$$A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 2 & 5 \\ 1 & 2 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 10 & 20 & 30 & 40 \\ 7 & 8 & 9 & -1 \\ 3 & 4 & 2 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 6 & 8 & 10 \end{bmatrix}$$

for matrix A $C_2 = 2C_1$, i.e the second column is twice of the first column. So columns are Linearly dependent

Thus: A^{-1} does not exist,

det $A = 0$

A is singular

rows are Linearly dependent

rank $A < 3$

For matrix B, $R_2 = 10R_1$ i.e rows are dependent

Thus

B^{-1} does not exist

B is singular

det $B = 0$

columns are dependent

rank $B < 4$

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For matrix C $R_3 = R_1 + R_2$, so rows are dependent

$\det C = 0$, C is singular, C^{-1} does not exists

columns are Linearly dependent

Note: it is not easy to see $R_3 = R_1 + R_2$

Reduce C into echolen form.

$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 6 & 8 & 10 \end{bmatrix} \xrightarrow[-6R_1 + R_3 \to R_3]{-5R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & -4 & -8 \end{bmatrix} \xrightarrow{-2R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & 0 & 0 \end{bmatrix}$$

So $\operatorname{rank} C = 2$ $\quad \operatorname{rank} C < 3 \Rightarrow \det C = 0$

C^{-1} does not exists

C is singular

Columns are dependent

7) Example problem it is known that

$$x = \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix} \quad z = \begin{bmatrix} 3 \\ 7 \\ 10 \end{bmatrix}$$ are Linearly dependent

ie there are nonzero a, b, c such that

$ax + by + cz = 0$. find a, b, c.

Solution.

$$a\begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix} + b\begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix} + c\begin{bmatrix} 3 \\ 7 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$a + 2b + 3c = 0$$
$$5a + 6b + 7c = 0$$
$$6a + 8b + 10c = 0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 6 & 8 & 10 \end{bmatrix} \xrightarrow[-6R_1 + R_3 \to R_3]{-5R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & -4 & -8 \end{bmatrix} \xrightarrow{-2R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -8 \\ 0 & 0 & 0 \end{bmatrix}$$

rank $A = 2$ multiple solution. $n = 3$ $r = 2$ $n - r = 3 - 2 = 1$

1 variable free. set $c = 1$. $-4b - 8c = 0 \Rightarrow b = -2$

$a + 2b + 3c = 0$ $\quad a + 2(-2) + 3(1) = 0 \Rightarrow a = +1$

Thus

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$$x - 2y + 3z = 0$$

Check

$$\begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix} - 2\begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix} + 1\begin{bmatrix} 3 \\ 7 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 - 4 + 3 \\ 5 - 12 + 7 \\ 6 - 16 + 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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