

4) Find the rank of the following matrices

$$a) A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}$$

$$b) B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$c) C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$d) D = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

$$e) E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$f) F = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 7 \end{bmatrix}$$

$$g) G = \begin{bmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 100 & 200 & 300 \end{bmatrix}$$

$$h) H = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

$$j) J = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 0 & 4 \\ 2 & 2 & 3 & 8 \end{bmatrix}$$

$$k) K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solutions

$$a) \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} \xrightarrow{\begin{array}{l} -\frac{5}{2}R_1 + R_2 \rightarrow R_2 \\ -4R_1 + R_3 \rightarrow R_3 \end{array}} \begin{bmatrix} 2 & 3 & 4 \\ 0 & -1.5 & -6 \\ 0 & -3 & -6 \end{bmatrix}$$

$$\xrightarrow{-2R_2 + R_3 \rightarrow R_3} \begin{bmatrix} 2 & 3 & 4 \\ 0 & -1.5 & -6 \\ 0 & 0 & 6 \end{bmatrix} \quad \text{Result Rank } A = 3$$

$$b) \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix} \xrightarrow{-2R_1 + R_2 \rightarrow R_2} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -2 & -4 & -6 \end{bmatrix}$$

$$\text{Rank } B = 2$$

or Rank 2

c) $C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ matrix is in echelon form N6
 $\text{rank } C = 3$

d) $D = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ $\begin{matrix} -2R_1 + R_2 \rightarrow R_2 \\ -3R_1 + R_3 \rightarrow R_3 \end{matrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\text{rank } D = 1$

e) $E = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\text{rank } E = 0$

f) $F = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 7 \end{bmatrix}$ $\text{rank } F = 3$

g) $G = \begin{bmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 100 & 200 & 300 \end{bmatrix}$ $\begin{matrix} -10R_1 + R_2 \rightarrow R_2 \\ -100R_1 + R_3 \rightarrow R_3 \end{matrix}$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\text{rank } G = 1$

h) $H = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ $R_2 \rightarrow R_2 - R_1$

method 1 The matrix is in Lower triangular form $\text{rank } H = 3$

method 2

$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ $\begin{matrix} -2R_1 + R_2 \rightarrow R_2 \\ -4R_1 + R_3 \rightarrow R_3 \end{matrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 5 & 6 \end{bmatrix}$ $-\frac{5}{3}R_2 + R_3 \rightarrow R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $\text{rank } H = 3$

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