Romazan Taşaltı EE300 N-150 A-1

Solve the following Linear Equations

$$\begin{aligned} 2x - 3y + 4z + w &= 2 \\ x + y + 3z + 4w &= 5 \\ 3x - 2y + 4z + 2w &= 1 \\ 7x + 6y + 10z + 6w &= 1.8 \end{aligned}$$

$$\tilde{A} = \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 1 & 1 & 3 & 4 & 5 \\ 3 & -2 & 4 & 2 & 1 \\ 7 & 6 & 10 & 6 & 1.8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 1 & 1 & 3 & 4 & 5 \\ 3 & -2 & 4 & 2 & 1 \\ 7 & 6 & 10 & 6 & 1.8 \end{bmatrix} \quad \begin{array}{l} -0.5R_1 + R_2 \to R_2 \\ -1.5R_1 + R_3 \to R_3 \\ -3.5R_1 + R_4 \to R_4 \end{array} \quad \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2.5 & 1 & 3.5 & 4 \\ 0 & 2.5 & -2 & 0.5 & -2 \\ 0 & 16.5 & -4 & 2.5 & -5.2 \end{bmatrix}$$

$$\begin{array}{l} -R_2 + R_3 \to R_3 \\ -\frac{16.5}{2.5}R_2 + R_4 \to R_4 \end{array} \quad \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2.5 & 1 & 3.5 & 4 \\ 0 & 0 & -3 & -3 & -6 \\ 0 & 0 & -10.6 & -20.6 & -31.6 \end{bmatrix}$$

$$-\frac{10.6}{3}R_3 + R_4 \to R_4 \quad \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2.5 & 1 & 3.5 & 4 \\ 0 & 0 & -3 & -3 & -6 \\ 0 & 0 & 0 & -10 & -10.4 \end{bmatrix}$$

$\text{rank } A = \text{rank } \tilde{A} = n$ (unique solution)

$$-10w = 10.4 \qquad w = \frac{-10.4}{-10} = \boxed{1.04}$$

$$-3z - 3w = -6$$

$$-3z - 3 \times 1.04 = -6 \qquad z = \frac{-6 + 3 \times 1.04}{-3} = \boxed{0.96}$$

$$-2.5y + z + 3.5w = 4$$

$$-2.5y + 0.96 + 3.5 \times 0.96 = 4 \qquad \boxed{y = -0.24}$$

$$2x - 3y + 4z + w = 2$$

$$2x - 3(-0.24) + 4(0.96) + 1.04 = 2 \qquad \boxed{x = -1.8}$$

R. A. T

solve the following Linear equations

$$2x - 3y + 4z + w = 2$$
$$x + y + 3z + 4w = 5$$
$$x - 4y + z - 3w = -3$$
$$5x + 7z + 4w = 5$$

$$\tilde{A} = \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 1 & 1 & 3 & 4 & 5 \\ 1 & -4 & 1 & -3 & -3 \\ 5 & 0 & 7 & 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 1 & 1 & 3 & 4 & 5 \\ 1 & -4 & 1 & -3 & -3 \\ 5 & 0 & 7 & 4 & 5 \end{bmatrix} \quad \begin{array}{l} -0{,}5R_1 + R_2 \to R_2 \\ -0{,}5R_1 + R_3 \to R_3 \\ -2{,}5R_1 + R_4 \to R_4 \end{array} \quad \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2{,}5 & 1 & 3{,}5 & 4 \\ 0 & -2{,}5 & -1 & -3{,}5 & -4 \\ 0 & 7{,}5 & -3 & 1{,}5 & 0 \end{bmatrix}$$

$$\begin{array}{l} R_2 + R_3 \to R_3 \\ -3R_2 + R_4 \to R_4 \end{array} \quad \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2{,}5 & 1 & 3.5 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & -9 & -12 \end{bmatrix} \to \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2{,}5 & 1 & 3.5 & 4 \\ 0 & 0 & -6 & -9 & -12 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Result rank $A = \text{rank}\,\tilde{A} = r = 3$ $\quad n = 4 \quad$ $n - r = 4 - 3 = 1$ free

set $\boxed{w = 0}$ $\quad -6z - 9w = -12 \Rightarrow -6z - 9 \times 0 = -12$ $\quad \boxed{z = 2}$

$2{,}5y + z + 3{,}5w = 4 \Rightarrow 2{,}5y + 2 + 3{,}5 \times 0 = 4$ $\quad \boxed{y = +0{,}8}$

$2x - 3y + 4z + w = 2 \Rightarrow 2x - 3 \times 0{,}8 + 4 \times 2 + 0 = 2$ $\quad \boxed{x = -1{,}8}$

---

set $w = 1$ $\quad -6z - 9w = -12 \quad -6z - 9 \times 1 = -12$ $\quad \boxed{z = 0{,}5}$

$2{,}5y + z + 3{,}5w = 4 \Rightarrow 2{,}5y + 0{,}5 + 3{,}5 = 4$ $\quad \boxed{y = 0}$

$2x - 3y + 4z + w = 2 \Rightarrow 2x - 3 \times 0 + 4 \times 0{,}5 + 1 = 2$ $\quad \boxed{x = -0{,}5}$

Dr Ramazan

Solve the following Linear equations.

$$\begin{aligned} 2x - 3y + 4z + w &= 2 \\ x + y + 3z + 4w &= 5 \\ -5y - 2z - 7w &= -8 \\ 3x - 2y + 7z + 5w &= 7 \end{aligned}$$

$$\tilde{A} = \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 1 & 1 & 3 & 4 & 5 \\ 0 & -5 & -2 & -7 & -8 \\ 3 & -2 & 7 & 5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 1 & 1 & 3 & 4 & 5 \\ 0 & -5 & -2 & -7 & -8 \\ 3 & -2 & 7 & 5 & 7 \end{bmatrix} \quad \begin{matrix} -0.5R_1 + R_2 \to R_2 \\ \\ -1.5R_1 + R_4 \to R_4 \end{matrix} \quad \begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2.5 & 1 & 3.5 & 4 \\ 0 & -5 & -2 & -7 & -8 \\ 0 & 2.5 & 1 & 3.5 & 4 \end{bmatrix}$$

$2R_2 + R_3 \to R_3$

$-R_2 + R_4 \to R_4$

$$\underbrace{\begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2.5 & 1 & 3.5 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{\hat{A}}$$

A

Note 1: rank A = rank $\tilde{A} = r = 2$

$n = 4$

$n - r = 4 - 2 = 2$ variables freely selected

We must use Echolen form equations

$$\begin{bmatrix} 2 & -3 & 4 & 1 & 2 \\ 0 & 2.5 & 1 & 3.5 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \to 2x - 3y + 4z + w = 2 \\ \to 2.5y + z + 3.5w = 4 \\ \\ \\ \end{matrix}$$

set $(z = 0,\ w = 1) \Rightarrow 2.5y + 0 + 3.5 \times 1 = 4$ $\quad y = 0.2$

$2x - 3y + 4z + w = 2 \Rightarrow 2x - 3 \times 0.2 + 4 \times 0 + 1 = 2$ $\quad x = 0.8$

Another solution: set $z = 1$ $\quad w = 0 \Rightarrow 2.5y + 1 + 3.5 \times 0 = 4$ $\quad y = 1.2$

$2x - 3y + 4z + w = 2 \Rightarrow 2x - 3 \times 1.2 + 4 \times 1 + 0 = 2$ $\quad x = 0.8$

Solve the following Linear Equations

$$2x - 3y + z + 2w = 5$$
$$6x - 9y + 3z + 6w = 15$$
$$10x - 15y + 5z + 10w = 25$$
$$8x - 12y + 4z + 8w = 20$$

$$\tilde{A} = \begin{bmatrix} 2 & -3 & 1 & 2 & 5 \\ 6 & -9 & 3 & 6 & 15 \\ 10 & -15 & 5 & 10 & 25 \\ 8 & -12 & 4 & 8 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 1 & 2 & 5 \\ 6 & -9 & 3 & 6 & 15 \\ 10 & -15 & 5 & 10 & 25 \\ 8 & -12 & 4 & 8 & 20 \end{bmatrix} \begin{matrix} -3R_1 + R_2 \rightarrow R_2 \\ -5R_1 + R_3 \rightarrow R_3 \\ -4R_1 + R_4 \rightarrow R_4 \end{matrix} \begin{bmatrix} \underbrace{\begin{matrix} 2 & -3 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}}_{A} & \begin{matrix} 5 \\ 0 \\ 0 \\ 0 \end{matrix} \end{bmatrix}$$

$\text{rank } A = \text{rank } \tilde{A} = r = 1$ $\quad n = 4$ multiple solution

$n - r = 4 - 1 = 3$ variables freely selected

solution $\quad 2x - 3y + z + 2w = 5$

set $x = 0 \quad y = 0 \quad z = 1 \rightarrow 1 + 2w = 5 \rightarrow w = 2$

set $x = 1 \quad y = 0 \quad z = 0 \rightarrow 2 + 2w = 5 \rightarrow w = 1.5$

set $x = 0 \quad y = 1 \quad z = 0 \rightarrow -3 + 2w = 5 \rightarrow w = 4$

$x = 1 \quad y = 1 \quad z = 0 \rightarrow 2 - 3 + 2w = 5 \rightarrow w = 6$

$x = 1 \quad y = 1 \quad z = 1 \rightarrow 2 - 3 + 1 + 2w = 5 \rightarrow w = 2.5$

$x = 10 \quad y = 10 \quad z = 10 \rightarrow 20 - 30 + 10 + 2w = 5 \rightarrow w = 2.5$

Dr Ramazan

B-1

Solve the following Linear homogenous system.

$$2x - 3y + 4z + w = 0$$
$$x + y + 3z + 4w = 0$$
$$3x - 2y + 4z + 2w = 0$$
$$7x - 6y + 10z + 6w = 0$$

$$A = \begin{bmatrix} 2 & -3 & 4 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & -2 & 4 & 2 \\ 7 & -6 & 10 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & -2 & 4 & 2 \\ 7 & -6 & 10 & 6 \end{bmatrix} \begin{matrix} -0.5R_1 + R_2 \to R_2 \\ -1.5R_1 + R_3 \to R_3 \\ -3.5R_1 + R_4 \to R_4 \end{matrix} \begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2.5 & 1 & 2 \\ 0 & 2.5 & -2 & 0.5 \\ 0 & 16.5 & -4 & 2.5 \end{bmatrix} \begin{matrix} -R_2 + R_3 \to R_3 \\ -\frac{16.5}{2.5}R_2 + R_4 \to R_4 \end{matrix} \begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2.5 & 1 & 3.5 \\ 0 & 0 & -3 & -3 \\ 0 & 0 & -10.6 & -20.6 \end{bmatrix}$$

$$-\frac{-10.6}{-3}R_3 + R_4 \to R_4 \quad \begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2.5 & 1 & 3.5 \\ 0 & 0 & -3 & -3 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

rank $A = n = 4$

No other solution exists except the trivial solution $x=0$ $y=0$ $z=0$ $w=0$

Dr Ramazan

Dr. Ramazan

Solve the following Linear Homogenous Equations

$$2x - 3y + 4z + w = 0$$
$$x + y + 3z + 4w = 0$$
$$x - 4y + z - 3w = 0$$
$$5x + 7z + 4w = 0$$

$$A = \begin{bmatrix} 2 & -3 & 4 & 1 \\ 1 & 1 & 3 & 4 \\ 1 & -4 & 1 & -3 \\ 5 & 0 & 7 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 \\ 1 & 1 & 3 & 4 \\ 1 & -4 & 1 & -3 \\ 5 & 0 & 7 & 4 \end{bmatrix} \begin{matrix} -0.5R_1 + R_2 \to R_2 \\ -0.5R_1 + R_3 \to R_3 \\ -2.5R_1 + R_4 \to R_4 \end{matrix} \qquad \begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2.5 & 1 & 3.5 \\ 0 & -2.5 & -1 & -3.5 \\ 0 & 7.5 & -3 & 1.5 \end{bmatrix} \begin{matrix} R_2 + R_3 \to R_3 \\ -3R_2 + R_4 \to R_4 \end{matrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2.5 & 1 & 3.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -6 & -9 \end{bmatrix} \begin{matrix} R_3 \to R_4 \\ R_4 \to R_3 \end{matrix} \begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2.5 & 1 & 3.5 \\ 0 & 0 & -6 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{result rank } A = r = 3$$

$n = 4$ rank $A = 3$ $n - r = 4 - 3 = 1$

There is one nontrivial independent solution

There is one variable free.

set $w = 1$ and solve for x, y, z

Note: It is straightforward to solve the <u>echolen form</u> equations.

$$\begin{bmatrix} 2 & -3 & 4 & 1 \\ 0 & 2.5 & 1 & 3.5 \\ 0 & 0 & -6 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 2x - 3y + 4z + w = 0 \\ -2.5y + z + 3.5w = 0 \\ -6z - 9w = 0 \end{matrix}$$

set $w = 1$ $\quad -6z - 9w = 0 \Rightarrow z = \frac{9}{-6} = -1.5$

$$-2.5y + z + 3.5w = 0 \Rightarrow -2.5y + (-1.5) + 3.5 \times 1 = 0 \Rightarrow y = -0.8$$

$$2x - 3y + 4z + w = 0 \Rightarrow 2x - 3(-0.8) + 4(-1.5) + 1 = 0 \Rightarrow x = 1.3$$

Dr R

Note: $x = 1.3$ $y = -0.8$ $z = -1.5$ $w = 1$ satisfies all 4 equations

$2x - 3y + 4z + w = 0 \Rightarrow 2(1.3) - 3(-0.8) + 4(-1.5) + 1 = 0$
$2.6 + 2.4 - 6 + 1 = 0$
$0 = 0$

$x + y + 3z + 4w = 0 \Rightarrow 1.3 + (-0.8) + 3(-1.5) + 4(1) = 0$
$1.3 - 0.8 - 4.5 + 4 = 0$
$0 = 0$

$x - 4y + z - 3w = 0$ $1.3 - 4(-0.8) + (-1.5) - 3(1) = 0$
$1.3 + 3.2 - 1.5 - 3 = 0$
$0 = 0$

$5x + 7z + 4w = 0$ $5(1.3) + 7(-1.5) + 4(1) = 0$
$+6.5 - 10.5 + 4 = 0$
$0 = 0$

Note: $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1.3 \\ -0.8 \\ -1.5 \\ 1 \end{bmatrix} \alpha$ is a solution. α is any number.

Example $\alpha = 10$ $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 13 \\ -8 \\ -15 \\ 10 \end{bmatrix}$

$2x - 3y + 4z + w = 0 \Rightarrow 2(13) - 3(-8) + 4(-15) + 10 = 0$
$26 + 24 - 60 + 10 = 0$
$0 = 0$

$x + y + 3z + 4w = 0 \Rightarrow 13 + (-8) + 3(-15) + 4(10) = 0$
$13 - 8 - 45 + 40 = 0$
$0 = 0$

$x - 4y + z - 3w = 0 \Rightarrow 13 - 4(-8) + (-15) - 3(10) = 0$
$13 + 32 - 15 - 30 = 0$
$0 = 0$

$5x + 7z + 4w = 0 \Rightarrow 5(13) + 7(-15) + 4(10) = 0$
$65 - 105 + 40 = 0$
$0 = 0$

solve the following Linear homogenous Equations

$$\begin{aligned} 2x-3y+4z+w&=0\\ x+y+3z+4w&=0\\ -5y-2z-7w&=0\\ 3x-2y+7z+5w&=0 \end{aligned} \qquad A=\begin{bmatrix} 2 & -3 & 4 & 1\\ 1 & 1 & 3 & 4\\ 0 & -5 & -2 & -7\\ 3 & -2 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1\\ 1 & 1 & 3 & 4\\ 0 & -5 & -2 & -7\\ 3 & -2 & 7 & 5 \end{bmatrix} \begin{matrix} -0.5R_1+R_2\rightarrow R_2\\ \\ -1.5R_1+R_4\rightarrow R_4 \end{matrix} \begin{bmatrix} 2 & -3 & 4 & 1\\ 0 & 2.5 & 1 & 3.5\\ 0 & -5 & -2 & -7\\ 0 & 2.5 & 1 & 3.5 \end{bmatrix} \begin{matrix} 2R_2+R_3\rightarrow R_3\\ -R_2+R_4\rightarrow R_4 \end{matrix}$$

$$\begin{bmatrix} 2 & -3 & 4 & 1\\ 0 & 2.5 & 1 & 3.5\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} \text{rank } A = r &= 2\\ n &= 4\\ n-r &= 4-2=2 \end{aligned}$$

There are two free variables

There are two independent nontrivial solution.

---

solution: Use Echolen form equations

$$\begin{bmatrix} 2 & -3 & 4 & 1\\ 0 & 2.5 & 1 & 3.5\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x\\ y\\ z\\ w \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix} \Rightarrow \begin{aligned} 2x-3y+4z+w&=0\\ 2.5y+z+3.5w&=0 \end{aligned}$$

set $z=0$ $w=1$

$2.5y+z+3.5w=0 \rightarrow 2.5y+0+3.5(1)=0$

$y=-1.4$

$2x-3y+4z+w=0 \rightarrow 2X-3(-1.4)+4(0)+1=0$

$X=-2.6$

---

set $z=1$ $w=0$

$2.5y+z+3.5w=0 \rightarrow 2.5y+1+3.5(0)=0$

$y=-0.4$

$2x-3y+4z+w=0 \rightarrow 2x-3(0.4)+4(1)+0=0$

$X=-2.6$

B5

Note: $V1 = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -2.6 \\ -1.4 \\ 0 \\ 1 \end{bmatrix}$ is a solution

$V2 = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -2.6 \\ -0.4 \\ 1 \\ 0 \end{bmatrix}$ is another solution

it is clear that V2 and V1 are independent. Because $\not\exists$ α satisfies $V2 = \alpha V1$

We have 2 independent solutions.

The following values are also solution

$V3 = 10\,V1 = \begin{bmatrix} -26 \\ -14 \\ 0 \\ 10 \end{bmatrix}$

$V4 = 20\,V2 = \begin{bmatrix} -26 \\ 4 \\ 10 \\ 0 \end{bmatrix}$

$V5 = V1 + V2 = \begin{bmatrix} -52 \\ -18 \\ 10 \\ 10 \end{bmatrix}$

$V6 = 3V1 + 4V2 = \begin{bmatrix} -18.2 \\ -5.8 \\ 4 \\ 3 \end{bmatrix}$

Take V6 for example $x = -18.2$ $y = -5.8$ $z = 4$ $w = 3$

$2x - 3y + 4z + w = 0 \rightarrow 2(-18.2) - 3(-5.8) + 4(4) + 3 = 0$

$-36.4 + 17.4 + 16 + 3 = 0$

$0 = 0$

$x + y + 3z + 4w = 0 \rightarrow -18.2 + (-5.8) + 3(4) + 4(3) = 0$

$-24 + 24 = 0$

$0 = 0$

$-5y - 2z - 7w = 0 \rightarrow -5(-5.8) - 2(4) - 7(3) = 0$

$29 - 8 - 21 = 0$

$0 = 0$

$3x - 2y + 7z + 5w = 0 \rightarrow 0 = 0$

Solve the following Linear homogenous equations

$$
\begin{aligned}
2x - 3y + z + 2w &= 0 \\
6x - 9y + 3z + 6w &= 0 \\
10x - 15y + 5z + 10w &= 0 \\
8x - 12y + 4z + 8w &= 0
\end{aligned}
\qquad
A = \begin{bmatrix} 2 & -3 & 1 & 2 \\ 6 & -9 & 3 & 6 \\ 10 & -15 & 5 & 10 \\ 8 & -12 & 4 & 8 \end{bmatrix}
$$

$$
\begin{bmatrix} 2 & -3 & 1 & 2 \\ 6 & -9 & 3 & 6 \\ 10 & -15 & 5 & 10 \\ 8 & -12 & 4 & 8 \end{bmatrix}
\begin{array}{l} -3R_1 + R_2 \to R_2 \\ -5R_1 + R_3 \to R_3 \\ -4R_1 + R_4 \to R_4 \end{array}
\begin{bmatrix} 2 & -3 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\quad \text{rank } A = 1
$$

$n = 4$ rank $A = r = 1$ $n - r = 4 - 1 = 3$

There are 3 free variables

There are 3 independent solutions

---

solutions We have only one equation

$$2x - 3y + z + 2w = 0$$

set $y = 0,\ z = 0 \quad w = 1 \quad \to \quad 2x - 3.(0) + 0 + 2 \times 1 = 0$

$x = -1$

$2x - 3 \times 0 + 1 + 2 \times 0 = 0$

set $y = 0 \quad z = 1 \quad w = 0$

$x = -0.5$

$2x - 3 \times 1 + 0 + 0 = 0$

set $y = 1 \quad z = 0 \quad w = 0$

$x = 1.5$

independent Solutions $V_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad V_2 = \begin{bmatrix} -0.5 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad V_3 = \begin{bmatrix} -1.5 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

---

other possible Solutions

$V_4 = V_1 + V_2 + V_3 = \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \qquad V_5 = 2V_1 + 3V_2 + 4V_3 = \begin{bmatrix} -9.5 \\ 4 \\ 3 \\ 2 \end{bmatrix}$

$V_6 = 3V_1 - 2V_2 + 5V_3$.

Dr Ramazan