Iki degiskenli fonksiyonlarda

$(p+q)^2 = p^2+2pq+q^2$

$(p+q)^3 = p^3 +3p^2q +3pq^2+ q^3$

$(p+q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 +q^4$

$$f(x,y) = f(a,b) + \frac{1}{1!}\left(f_X(a,b)(x-a) + f_Y(a,b)(y-b) \right)$$
$$+\frac{1}{2!}\left(f_{XX}(a,b)(x-a)^2 + 2f_{XY}(a,b)(x-a)(y-y_0) + f_{YY}(a,b)(y-b)^2 \right)$$
$$+\frac{1}{3!}\left(f_{XXX}(a,b)(x-a)^3 + 3f_{XXY}(a,b)(x-a)^2(y-y_0) + 3f_{XYY}(a,b)(x-a)(y-y_0)^2 + f_{YYY}(a,b)(y-b)^3 \right)$$
$$+\frac{1}{4!}(f_{XXXX}(a,b)(x-a)^4 + \dots\dots\dots$$
$$+\dots\dots\dots$$

312) $f(x,y) = e^{x+y}$ ifadesini (0,0) civarinda Taylor serisine acin.
Cozum
$f_X = =e^{x+y}$, $f_{XX} = =e^{x+y}$, $f_{XXX} = =e^{x+y}$, ...
$f_Y = =e^{x+y}$, $f_{YY} = =e^{x+y}$, $f_{YY} = =e^{x+y}$, ...
$f_{XY} = =e^{x+y}$, $f_{XYX} = =e^{x+y}$, $f_{XYXY} = =e^{x+y}$, ...

$f(a,b)=e^{0+0} =1$, $f_X(a,b)=e^{0+0} =1$, $f_{XX}(a,b)=1$, $f_{XXX}=1$, ...

Bu ifadeler ana formulde yerinekonulursa

$$f(x,y) = 1\frac{1}{1!}\left(1(x-0) + (y-0)\right)$$
$$+\frac{1}{2!}\left(1(x-0)^2 + 2\ 1(x-0)(y-0) + 1\ (y-0)^2\right)$$
$$+\frac{1}{3!}\left(1(x-0)^3 + 3\ 1(x-0)^2(y-0) + 3\ (x-0)(y-0)^2 + (y-0)^3\right)$$
$$+\dots\dots\dots$$
$$= 1 + x + y + \frac{x^2+2xy+y^2}{2!} + \frac{x^3+3x^2y+3xy^2+y^3}{3!} + \dots.$$

322) f(x,y)=sin(xy) ifadesini (0,0) civarinda Taylor serisine acin.
Cozum:

f(a,b)=sin(0 0)=0

$f_X(x,y) = y\cos(xy)$, $f_X(0,0) = 0\cos(0\ 0)=0\ 1=0$

$f_Y(x,y) = x\cos(xy)$, $f_Y(0,0) = 0\cos(0\ 0)=0\ 1=0$

$f_{XX}(x,y) = -y^2 \sin(xy)$, $f_{XX}(0,0) = -0\sin(0\ 0)=0$

$f_{YY}(x,y) = -x^2 \sin(xy)$, $f_{XX}(0,0) = -0\sin(0\ 0)=0$

$f_{XY}(x,y) = -(1\ \cos(xy) - y\ y\sin(xy))$, $f_{XY}(0,0) =1$

....

....

$f_{XYX}(x,y) = -\ y\cos(xy) + ...$

.....

$f_{XYXY}(x,y) = -\ 1\cos(xy) + ...$ $= -1$

....

Bu degerler ana formulde yerlerine konulursa

$$f(x,y) = 0 + \frac{1}{1!}(0+0) + \frac{1}{2!}(0 + 2\ (x-0)(y-0) + 0) + ...$$

$$= \frac{2xy}{2!} - \frac{x^3y^3}{3!} + -\frac{x^5y^5}{5!} - ...$$

Sonuc: $$\sin(xy) = xy - \frac{(xy)^3}{3!} + \frac{(xy)^5}{5!} - \frac{(xy)^7}{7!}...$$

322) f(x,y)=cos(xy) ifadesini (0,0) civarinda Taylor serisine acin.
Cozum:

f(a,b)=cos(0 0)=cos(0)=1

$f_X(x,y) = -y\sin(xy)$, $f_X(0,0) = -0\sin(0\ 0)=0$

$f_Y(x,y) = -x\sin(xy)$, $f_Y(0,0) = -0\sin(0\ 0)=0$

$f_{XX}(x,y) = -y^2 \cos(xy)$, $f_{XX}(0,0) = -0\cos(0\ 0)=0$

$f_{YY}(x,y) = -x^2 \cos(xy)$, $f_{XX}(0,0) = -0\cos(0\ 0)=0$

$f_{XY}(x,y) = -(1\ \sin(xy) + y\ y\cos(xy))$, $f_{XY}(0,0) =0$

$$\cos(xy) = 1 - \frac{(xy)^2}{2!} + \frac{(xy)^4}{4!} - \frac{(xy)^6}{6!}...$$

ÖRNEK 1 Kuadratik Bir Yaklaşım Bulmak

$f(x, y) = \sin x \sin y$'ye orijin civarında kuadratik bir yaklaşım bulun. $|x| \leq 0.1$ ve $|y| \leq 0.1$ ise, yaklaşım ne kadar kesindir?

Çözüm (8) Denklemlerinde $n = 2$ alırız:

$$f(x, y) = f(0, 0) + (xf_x + yf_y) + \frac{1}{2}(x^2 f_{xx} + 2xyf_{xy} + y^2 f_{yy})$$

$$+ \frac{1}{6}(x^3 f_{xxx} + 3x^2 y f_{xxy} + 3xy^2 f_{xyy} + y^3 f_{yyy})_{(cx,cy)}$$

$$f(0, 0) = \sin x \sin y|_{(0,0)} = 0, \qquad f_{xx}(0, 0) = -\sin x \sin y|_{(0,0)} = 0$$

$$f_x(0, 0) = \cos x \sin y|_{(0,0)} = 0, \qquad f_{xy}(0, 0) = \cos x \cos y|_{(0,0)} = 1.$$

$$f_y(0, 0) = \sin x \cos y|_{(0,0)} = 0, \qquad f_{yy}(0, 0) = -\sin x \sin y|_{(0,0)} = 0$$

$$\sin x \sin y \approx 0 + 0 + 0 + \frac{1}{2}(x^2(0) + 2xy(1) + y^2(0)),$$

$$\sin x \sin y \approx xy$$