Vektorlerin lineer bagimsizligi

Ornek 211

x + 2y = 10,
2x + 4y = 20 **Denklem Takimini Cozun**

x + 2y = 10 => x=10-2y
Ikinci denklemde yerine koy
2(10-2y) + 4y = 20
20-4y)+ 4y = 20
20=20
Sonuc:
x+2y=10 => x=10-2y sartini saglayan butun x ve y ler her iki denklemi de cozer.
(x=0, y=5), (x=2, y=4), (x=4, y=3), ...
Denklem takiminin sonsuz cozumu vardir.

Ornek 212

x + 2y = 10,
2x + 4y = 30 **Denklem Takimini Cozun**

x + 2y = 10 => x=10-2y
Ikinci denklemde yerine koy
2(10-2y) + 4y = 30
20-4y)+ 4y = 30
20=30
Her iki denklemi cozecek x ve y degerleri mevcut degildir. Denklem takimi Cozumsuz.

Ornek 213

x + 2y = 10,
2x + 3y = 19 **Denklem Takimini Cozun**

x + 2y = 10 => x=10-2y
Ikinci denklemde yerine koy
2(10-2y) + 3y = 19
20-4y+ 3y = 19
-y=-20+19
y=1
x=10-2y=8
Denklemin cozumu tekdir. (x=8, y=1) denkelmin tek cozumudur.

Bir lineer denklem takiminda katsayilar matrisi lineer bagimli ise cozum ya yoktur veya sonsuz coxum vardir.
Katsayilar matrisi ornek 211 ve 212 de
1.satir - -> 1 2
2. satir -->2 4
Birinci satiri -2 ile carpip 2. satira eklesek sifir olur. Yani birinci ve ikinci satir arasinda bir baginti vardir.

Bu durumda ya cozum yoktur veya sonsuz cozum vardir. KIsaca cozum tek degildir. Veya cozumler birbirine lineer bagimlidir.

Vektorlerin Dogrusal Bagimsizligi
Linear independence of vectors

1) p=[1 5 3], p is a 3 dimensional **row** vector
p 3 elemanli satir vektordur. p nin boyutu 3 dur.

$q=\begin{bmatrix}3\\1\end{bmatrix}$, q is a 2 dimensional **column** vector
q 2 elemanli sutun vektordur. q nin boyutu 2 dir.

Definition: $X_1, X_2, X_3, \ldots, X_n$, are vectors
If there are numbers $a_1, a_2, a_3, \ldots a_n$, such that
$a_1X_1+a_2X_2+a_3X_3+\ldots+a_nX_n=0$
then vectors $X_1, X_2, X_3, \ldots, X_n$ are linearly dependent.

For two vectors,
$a_1X_1+a_2X_2=0$
or
$a_1 X_1 = -a_2 X_2$

$X_1 = -\frac{a_2}{a_1} X_2 = \alpha X_2$

i.e. if the two vectors are multiples of each other then the two vectors are linearly dependent

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2) $m=\begin{bmatrix}1\\3\\2\end{bmatrix}$, $n=\begin{bmatrix}10\\30\\20\end{bmatrix}$

It is clear that n=10m or m=0.1n. Thus m and n are linearly dependent

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3) $m=\begin{bmatrix}1\\3\end{bmatrix}$, $n=\begin{bmatrix}2\\1\end{bmatrix}$

There is no number α that satisfies the equation n= α m. Thus m and n are linearly independent

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4) $m=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$, $n=\begin{bmatrix}0\\0\\0\\4\end{bmatrix}$

It is clear that n=4m. m and n are linearly dependent

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5) $m=\begin{bmatrix}1\\3\\-1\\10\end{bmatrix}$, $n=\begin{bmatrix}-10\\-30\\10\\-100\end{bmatrix}$

n=-10m. m and n are linearly dependent

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6) $m=\begin{bmatrix}0\\2\\1\end{bmatrix}$, $n=\begin{bmatrix}0\\0\\3\end{bmatrix}$

There is no number α that satisfies the eqation n= αm
m and n are linearly independent.

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6) $m=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $n=\begin{bmatrix}0\\5\\0\end{bmatrix}$, $p=\begin{bmatrix}8\\0\\0\end{bmatrix}$

am+bn+cp=0

$a\begin{bmatrix}0\\0\\1\end{bmatrix} + b\begin{bmatrix}0\\5\\0\end{bmatrix} + c\begin{bmatrix}8\\0\\0\end{bmatrix} = 0$

a 0 +b 0+c 8=0 → c 8=0 → c=0
a 0 +b 5+c 0=0 → b 5=0 → b=0
a 1 +b 0+c 0=0 → a 1=0 → a=0

There is no numbers (except zero) that satisfies the equation am+bn+cp=0. Thus the vectors m,n,p are linearly independent.

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7) $m=\begin{bmatrix}0\\8\\2\end{bmatrix}$, $n=\begin{bmatrix}0\\0\\3\end{bmatrix}$, $p=\begin{bmatrix}4\\5\\6\end{bmatrix}$

am+bn+cp=0

$a\begin{bmatrix}0\\8\\2\end{bmatrix} + b\begin{bmatrix}0\\0\\3\end{bmatrix} + c\begin{bmatrix}4\\5\\6\end{bmatrix} = 0$

a 0 +b 0+c 4=0 → c 4=0 → c=0
a 8 +b 0+c 5=0 → 8a + 5c=0 → 8a + 5 0=0 →a =0
a 2 +b 3+c 6=0 → 2a+0+0=0 → a=0

There are no numbers (except a=0, b=0, c=0) that satisfies the equation am+bn+cp=0. Thus the vectors m,n,p are linearly independent.

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7) $m=\begin{bmatrix}1\\3\\2\end{bmatrix}$, $n=\begin{bmatrix}2\\6\\4\end{bmatrix}$, $p=\begin{bmatrix}0\\0\\1\end{bmatrix}$

$a\begin{bmatrix}1\\3\\2\end{bmatrix} + b\begin{bmatrix}2\\6\\4\end{bmatrix} + c\begin{bmatrix}0\\0\\1\end{bmatrix} = 0$

a 1 +b 2+c 0=0 → a+2b=0 → a=-2b
a 3 +b 6+c 0=0 → 3a + 6b=0 → a=-2b
a 2 +b 4+c 1=0 → 2a+4b+c=0 → replace a=-2b
2(-2b)+ 4b+c=0 → -4b+4b +c=0 → c=0

The values c=0, a=-2b satisfies the equation.

Example c=0, b=1, a=-2 or c=0, b=10, a=-20
etc satisfies the equation am+bn+cp=0
Vectors are linearly dependent.

LINEER DENKLEMLER (Denklem sayisi= Bilinmeyen sayisi)

$$\begin{matrix} x+y=4 \\ 2x-y=5 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 2 & -1 & 5 \end{bmatrix} \xrightarrow{R_2-2R_1 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 4 \\ 0 & -3 & -3 \end{bmatrix} \rightarrow \text{Tek Cozum} \rightarrow \begin{matrix} y=2 \\ x=1 \end{matrix}$$

$$\begin{matrix} x+y+z=7 \\ 4x-3y+z=6 \\ 2x-7y-2z=-9 \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 7 \\ 4 & -3 & 1 & 6 \\ 2 & -7 & -2 & -9 \end{bmatrix} \xrightarrow[R_3-2R_1 \rightarrow R_3]{R_2-4R_1 \rightarrow R_2} \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -11 & -3 & -22 \\ 0 & -11 & -4 & -23 \end{bmatrix} \xrightarrow{R_3-R_2 \rightarrow R_3}$$

$$\begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -11 & -3 & -20 \\ 0 & 0 & -1 & -3 \end{bmatrix} \rightarrow \text{Tek Cozum} \rightarrow \begin{matrix} z=3 \\ y=1 \\ x=2 \end{matrix}$$

$$\begin{matrix} x+y=4 \\ 2x+2y=8 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 8 \end{bmatrix} \xrightarrow{R_2-2R_1 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Sonsuz Cozum}$$

$$\begin{matrix} x+y+z=7 \\ 4x-3y+z=6 \\ 2x-7y-z=-8 \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 7 \\ 4 & -3 & 1 & 6 \\ 2 & -7 & -1 & -8 \end{bmatrix} \xrightarrow[R_3-2R_1 \rightarrow R_3]{R_2-4R_1 \rightarrow R_2} \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -11 & -3 & -22 \\ 0 & -11 & -3 & -22 \end{bmatrix} \xrightarrow{R_3-R_2 \rightarrow R_3}$$

$$\begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -11 & -3 & -22 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Sonsuz Cozum}$$

$$\begin{matrix} x+y=4 \\ 2x+2y=6 \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 6 \end{bmatrix} \xrightarrow{R_2-2R_1 \rightarrow R_2} \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & -2 \end{bmatrix} \rightarrow \text{Cozum yok}$$

$$\begin{matrix} x+y+z=7 \\ 4x-3y+z=6 \\ 2x-7y-z=-5 \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 7 \\ 4 & -3 & 1 & 6 \\ 2 & -7 & -1 & -5 \end{bmatrix} \xrightarrow[R_3-2R_1 \rightarrow R_3]{R_2-4R_1 \rightarrow R_2} \begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -11 & -3 & -22 \\ 0 & -11 & -3 & -19 \end{bmatrix} \xrightarrow{R_3-R_2 \rightarrow R_3}$$

$$\begin{bmatrix} 1 & 2 & 1 & 7 \\ 0 & -11 & -3 & -22 \\ 0 & 0 & 0 & 3 \end{bmatrix} \rightarrow \text{Cozum yok}$$

Ozet

$$\begin{bmatrix} \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ 0 \ldots\ldots 0 \quad X \quad X \end{bmatrix} \rightarrow \text{Tek Cozum} \qquad \begin{bmatrix} \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ 0 \quad 0 \ldots 0 \quad 0 \end{bmatrix} \rightarrow \text{Sonsuz Cozum} \qquad \begin{bmatrix} \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ 0 \quad 0 \ldots 0 \quad X \end{bmatrix} \rightarrow \text{Cozum Yok}$$

Genisletilmis matris Gaus eliminasyon yontemiyle alt ucgeni sifirlamaya calis.
Eger en son satir tamamen sifir ise cozum sonsuz tane
Eger en son satirin en son 2 elemani sifirdan farkli ise tek cozum
Eger en son satirin sadece en son 1 elemani sifirdan farkli ise tek cozum yok

$$\begin{aligned} x+y+2z+3w&=1\\ x+y+3z+5w&=4\\ 2x+5y+8z+11w&=6\\ -x+2y+4z+4w&=4 \end{aligned} \rightarrow \begin{bmatrix} 1&1&2&3&1\\ 1&1&3&5&4\\ 2&5&8&11&6\\ -1&2&4&4&4 \end{bmatrix} \xrightarrow{\substack{R_2-R_1\to R_2\\ R_3-2R_1\to R_3\\ R_4-R_1\to R_4}} \begin{bmatrix} 1&1&2&3&1\\ 0&0&1&2&3\\ 0&3&4&5&4\\ 0&3&6&7&5 \end{bmatrix}$$

$$\begin{bmatrix} 1&1&2&3&1\\ 0&0&1&2&3\\ 0&3&4&5&4\\ 0&3&6&7&5 \end{bmatrix} \xrightarrow{\substack{R_2\to R_3\\ R_3\to R_2}} \begin{bmatrix} 1&1&2&3&1\\ 0&3&4&5&4\\ 0&0&1&2&3\\ 0&3&6&7&5 \end{bmatrix} \xrightarrow{R_4-R_2\to R_4} \begin{bmatrix} 1&1&2&3&1\\ 0&3&4&5&4\\ 0&0&1&2&3\\ 0&0&2&2&1 \end{bmatrix}$$

$$\begin{bmatrix} 1&1&2&3&1\\ 0&3&4&5&4\\ 0&0&1&2&3\\ 0&0&2&2&1 \end{bmatrix} \xrightarrow{R_4-2R_3\to R_4} \begin{bmatrix} 1&1&2&3&1\\ 0&3&4&5&4\\ 0&0&1&2&3\\ 0&0&0&-2&-5 \end{bmatrix} \rightarrow \text{Unique Solution}$$

w=2.5
z=-2
y=-0.1667
x=-2.333

$$\begin{bmatrix} 1&2&2&3&1\\ 2&4&3&5&4\\ 1&2&6&7&10\\ 3&6&8&10&12 \end{bmatrix} \xrightarrow{\substack{R_2-2R_1\to R_2\\ R_3-R_1\to R_3\\ R_4-3R_1\to R_4}} \begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&4&4&9\\ 0&0&2&1&9 \end{bmatrix} \xrightarrow{\substack{R_3+4R_2\to R_3\\ R_4+2R_2\to R_4}} \begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&0&0&17\\ 0&0&0&-1&13 \end{bmatrix}$$

$$\begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&0&0&17\\ 0&0&0&-1&13 \end{bmatrix} \xrightarrow{\substack{R_2\to R_3\\ R_3\to R_2}} \begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&0&-1&13\\ 0&0&0&0&17 \end{bmatrix} \rightarrow \text{No Solution}$$

$$\begin{bmatrix} 1&2&2&3&1\\ 2&4&3&5&4\\ 1&2&6&7&-7\\ 3&6&8&10&12 \end{bmatrix} \xrightarrow{\substack{R_2-2R_1\to R_2\\ R_3-R_1\to R_3\\ R_4-3R_1\to R_4}} \begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&4&4&-8\\ 0&0&2&1&9 \end{bmatrix} \xrightarrow{\substack{R_3+4R_2\to R_3\\ R_4+2R_2\to R_4}} \begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&0&0&0\\ 0&0&0&-1&13 \end{bmatrix}$$

$$\begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&0&0&0\\ 0&0&0&-1&13 \end{bmatrix} \xrightarrow{\substack{R_2\to R_3\\ R_3\to R_2}} \begin{bmatrix} 1&1&2&3&1\\ 0&0&-1&-1&2\\ 0&0&0&-1&13\\ 0&0&0&0&0 \end{bmatrix} \rightarrow \text{Multiple Solution}$$

A: katsayilar matrisi

$\tilde{A}$ **: Genisletilmis matris**.

r : bilinmeyen sayisi. (nxn icin denklem sayisi)

rank A=rank $\tilde{A}$=r ==>tek cozum

rank A=rank $\tilde{A}$ < r ==>sonsuz cozum

rank A< rank $\tilde{A}$==> cozum yok

$$A=\begin{vmatrix} 2 & 4 & 2 & 0 & 8 \\ 0 & 4 & 0 & 1 & 10 \\ 0 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 14 & 4 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & 4 & 2 & 0 & 8 \\ 0 & 4 & 0 & 1 & 10 \\ 0 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 7 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Rank A=rank $\tilde{A}$=r=4 **Tek Cozum**

$$A=\begin{vmatrix} 2 & 4 & 2 & 0 & 8 \\ 0 & 4 & 2 & 1 & 10 \\ 0 & 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 14 & 4 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & 4 & 2 & 0 & 8 \\ 0 & 4 & 2 & 1 & 10 \\ 0 & 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 0 & -22 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Rank A=3, rank $\tilde{A}$=4 **Cozum yok**

$$A=\begin{vmatrix} 2 & 4 & 2 & 0 & 8 \\ 0 & 4 & 2 & 1 & 10 \\ 0 & 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 4 & 16 \\ 0 & 0 & 0 & 20 & 80 \end{vmatrix} \rightarrow \begin{vmatrix} 2 & 4 & 2 & 0 & 8 \\ 0 & 4 & 2 & 1 & 10 \\ 0 & 0 & 0 & 2 & 8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

rank A=rank $\tilde{A}$ =3<**4** **Sonsuz cozum**

Find the solutions for the equations

x+2y- 3z +4w=10, 2x+5y-z+9w=5

The Augmented matrix is

1 2 -3 4 10
2 5 -1 9 5

The **echelon** form is

1 2 -3 4 10
0 1 5 1 -15

n=4, rank A=rank $\tilde{A}$=2<4 multiple solution

n-r=4-2= 2 variables are arbitrarily selected. (2 free variables)

x+2y- 3z +4w=10, y+5z+w=-15

First solution

Set z=0 w=1 and solve for x and y:

x+2y- 3 0 +4 1=10, y+5 0+ 1=-15

y=-16, x+2y=6 → x=38

V1=[x y z w]=[38 -16 0 1]

Second solution

Set z=1 w=0 and solve for x and y:

x+2y- 3 1 +4 0=10, y+5 1+ 0=-15

y=-20, x+2y=13 → x=53

V2=[x y z w]=[53 -20 1 0]

Third Solution

Set y=0, z=0 and solve for x and y:

x+2 0- 3 0 +4w=10, 0 +5 0+w=-15

w=-15, x+4w=10, → x=70

V3=[x y z w]=[70 0 0 -15]

Forth Solution

Set y=10, z=20

x+2 10- 3 20 +4w=10, 10 +5 20+w=-15

w=-125; x+4w=50; → x=550

V4=[550 10 20 -125]

Find the solution space for the homogenous system

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & -1 \end{bmatrix}$$

since rank A=n the system has empty solution space. (**no solution exists** except the trivial solution x=0 y=0 z=0)

Find the solution space for the homogenous system

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

rank A=r=2 n=3; n-r=3-2=1

The system has **one nontrivial** solution. To find that solution **set z=1;** and use equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \Rightarrow \begin{matrix} x+2y+3z=0 \\ -3y-6z=0 \end{matrix} \Rightarrow \begin{matrix} x+2y+3=0 \\ -3y-6=0 \end{matrix}$$ y=-2; x=1;

Solution set is **V1**=[x y z]=[1 -2 1]

There is only one (n-r=1) independent solution. All other solutions will be multiples of this single solution. For example **set z=10**;

$$\begin{matrix} x+2y+3z=0 \\ -3y-6z=0 \end{matrix} \Rightarrow \begin{matrix} x+2y+30=0 \\ -3y-60=0 \end{matrix} \Rightarrow \begin{matrix} x+2y=-30 \\ -3y=60 \end{matrix} \Rightarrow \begin{matrix} x=30 \\ y=-20 \end{matrix}$$

V2=[x y z]=[10 -20 10]

It is clear that V2=10*V1. i.e V1 and V2 are linearly dependent.

Find the solution space for the homogenous system

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 8 & 12 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

rank A=r=1 n=3; n-1=3-1=2

2 variables are freely selected.

Set y=0,z=1 calculate x. (x=-3). **V1=[-3 0 1].**

Set y=1, z=0⇒ x=-2 **V2=[-2 1 0]**

All other solutions will be linearly dependent.

Gaus Eliminasyon Yontemiyle Lineer denklemlerin cozumu

$$\begin{bmatrix} 2 & 6 & 8 \\ 4 & 17 & 14 \\ 1 & 13 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 30 \end{bmatrix} \quad \underset{\Rightarrow}{\text{Augmented Matrix}} \quad \begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix}$$

Birinci satiri 2 ile carp ikinci satira ilave et.

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix} \qquad R_2 - 2R_1 \rightarrow R_2$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4-2*2 & 17-2*6 & 14-2*8 & 15-2*4 \\ 1 & 13 & 14 & 30 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 1 & 13 & 14 & 30 \end{bmatrix}$$

Birinci satiri -0.5 ile carp ucuncu satira ilave et.

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 1 & 13 & 14 & 30 \end{bmatrix} \qquad R_3 - 0.5R_1 \rightarrow R_3$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 1-0.5*2 & 13-0.5*6 & 14-0.5*8 & 3-0.5*4 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10 & 10 & 30 \end{bmatrix}$$

Ikinci satiri -2 ile carp ikinci satira ilave et.

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10 & 10 & 28 \end{bmatrix} \quad R_3 - 2R_2 \rightarrow R_3$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10-2*5 & 10-2*(-2) & 28-2*7 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

Sonuc denklemleri yaz.

$$2x + 6y + 8z = 4$$
$$5y - 2z = 17$$
$$14z = 14$$

Sondan Baslayarak denklemleri coz.

$$14z = 14 \quad \Rightarrow \quad z = \frac{14}{14} = 1$$

$$5y - 2z = 7 \quad \Rightarrow \quad 5y - 2*1 = 7 \quad \Rightarrow \quad 5y = 9 \quad \Rightarrow \quad y = \frac{9}{5} = 1.8$$

$$2x + 6y + 8z = 4 \quad \Rightarrow \quad 2x + 6*1.8 + 8*1 = 4 \quad \Rightarrow \quad 2x + 10.8 + 8 = 4 \quad \Rightarrow \quad x = -7.4$$

During the process we converted the coefficient matrix into **upper triangular Form**

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 2 & 6 & 6 & 4 \\ 0 & 5 & -7 & 1 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

Aciklama

$2x+6y+8z=4 \qquad (E1)$

$4x+17y+14z=15 \qquad (E2)$

$x+13y+14z=30 \qquad (E3)$

Birinci esitligin (E1) iki tarafini -2 ile carp (E1)

$2x+6y+8z=4 \quad \Rightarrow -4x-12y-16z=-8$

Elde edilen bu yeni esitligi ikinci esitlige (E2) ekle.

$$\begin{array}{rl} & -4x-12y-16z=-8 \\ + & \;\;4x+17y+14z=\;\;15 \\ \hline & \qquad 5y-2z=7 \qquad (E4) \end{array}$$

Multiply both side of equation (E1) by –0.5.

$2x+6y+8z=4 \quad \Rightarrow \quad -x-3y-4z=-2$

Add this modified equation (E1) to equation (E3).

$$\begin{array}{rl} & -x-3y\;\;-4z=-2 \\ + & \;\;x+13y+14z=\;\;30 \\ \hline & 10y+10z\;=\;28 \qquad (E5) \end{array}$$

Now Solve (E4) and (E5)

$5y-2z=7 \qquad (E4)$

$10y+10z=28 \qquad (E5)$

Multiply equation (E4) by –2 and add to equation (E5)

$5y-2z=7 \quad \Rightarrow \quad -10y+4z=-14$

$$\begin{array}{rl} & -10y+4z\;\;=-14 \\ + & \;\;10y\;\;+10z\;\;=28 \\ \hline & \qquad 14z=14 \end{array}$$

Thus $z=1$. Substitute z=1 into equation (E4) and obtain y.

$5y-2z=7 \quad \Rightarrow \quad 5y-2*1=7$

$5y=9 \quad \Rightarrow \quad y=\frac{9}{5}=1.8$

Substitute z=1 and y=1.8 into the first equation of (E1) and solve for x.

$2x+6y+8z=4 \quad \Rightarrow \quad 2x+6*1.8+8*1=4$

$\Rightarrow \quad 2x+10.8+8=4 \quad \Rightarrow \quad x=-7.4$

Result x=-7.4 y=1.8 z=1

Gaus Jordan (diagonal kosegen form) Yontemiyle Denklem Cozumu

Hedef
$$\begin{bmatrix} X & X & X & X & X & X \\ X & X & X & X & X & X \\ X & X & X & X & X & X \\ X & X & X & X & X & X \\ X & X & X & X & X & X \end{bmatrix} \Longrightarrow \begin{bmatrix} X & X & X & X & X & X \\ 0 & X & X & X & X & X \\ 0 & 0 & X & X & X & X \\ 0 & 0 & 0 & X & X & X \\ 0 & 0 & 0 & 0 & X & X \end{bmatrix} \Longrightarrow \begin{bmatrix} X & 0 & 0 & 0 & 0 & X \\ 0 & X & 0 & 0 & 0 & X \\ 0 & 0 & X & 0 & 0 & X \\ 0 & 0 & 0 & X & 0 & X \\ 0 & 0 & 0 & 0 & X & X \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 8 \\ 4 & 17 & 14 \\ 1 & 13 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 30 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix} \quad \begin{matrix} -2R_1 + R_2 \to R_2 \\ -0.5R_1 + R_3 \to R_3 \end{matrix} \quad \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10 & 10 & 28 \end{bmatrix} \quad -2R_2 + R_3 \to R_3 \quad \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 0 & 14 & 14 \end{bmatrix} \quad \begin{matrix} -\left(\frac{-2}{14}\right)R_3 + R_2 \to R_2 \\ -\left(\frac{8}{14}\right)R_3 + R_1 \to R_1 \end{matrix} \qquad \begin{matrix} \frac{1}{7}R_3 + R_2 \to R_2 \\ -\frac{6}{7}R_3 + R_1 \to R_1 \end{matrix}$$

$$\begin{bmatrix} \frac{1}{7}\text{x}0+2 & \frac{1}{7}\text{x}0+6 & \frac{1}{7}\text{x}14+8 & \frac{1}{7}\text{x}14+4 \\ -\frac{6}{7}\text{x}0+0 & -\frac{6}{7}\text{x}0+5 & -\frac{6}{7}\text{x}14-2 & -\frac{6}{7}\text{x}14+7 \\ 0 & 0 & 14 & 14 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 0 & -4 \\ 0 & 5 & 0 & 9 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

$$-\frac{6}{5}R_2 + R_1 \to R_1 \begin{bmatrix} -\frac{6}{5}\text{x}0+2 & -\frac{6}{5}\text{x}5+6 & -\frac{6}{5}\text{x}0+0 & -\frac{6}{5}\text{x}9-4 \\ 0 & 5 & 0 & 9 \\ 0 & 0 & 14 & 14 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -14.8 \\ 0 & 5 & 0 & 9 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

2x=-14.8 $\Rightarrow$ x=-7.4, 5y=9 $\Rightarrow$ y=1.8, 14z=14 $\Rightarrow$ z=1